History of RSB Interview: David Ruelle

June 13-20, 2021. Final revision: June 20, 2021

Interviewers:

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Location:

Assembled from a series of email exchanges over the interview period.

PC: [This interview focuses on the topics of spin glasses and replica symmetry breaking. Because many aspects in your life and career were already covered in prior interviews you have given[1], let's jump directly into the topic.] How did you first hear about spin glasses and replica symmetry breaking?

DR: The first contact I remember with replica symmetry breaking was through Parisi's papers on the Parisi ansatz.

PC: In your 1987 Comm. Math. Phys. about the REM and the GREM, you thank Nicolas Sourlas, Bernard Derrida and Cirano de Dominicis for "much needed guidance in the mystifying field of Parisi's Ansatz, REM and GREM."[2] From your perspective, what was mystifying about these topics? And what guidance did you receive from these researchers?

DR: My mystification was with the mathematics and the help I received was with the physics.

        What I remember from replica symmetry breaking is that prior to my contribution on Derrida's REM and GREM, I had been perplexed by Parisi's

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[1] Hans Henrik Rugh, "Un interview de David Ruelle," *La Gazette des Mathématiciens*, **161**(July) 55-59 (1999). https://smf.emath.fr/publications/la-gazette-des-mathematiciens-161-juillet-2019 (Consulted June 20, 2021); "An Interview with David Ruelle", *European Mathematical Society Newsletter*, **115**(March), 21-24 (2021). https://doi.org/10.4171/NEWS/115/6. See also: Anne-Sandrine Paumier, "Mémoires de David Ruelle, entretien du 2 avril 2015 - The Golden Years of the IHES," *Institut des Hautes Études Scientifiques (IHÉS) Youtube Channel,* Feb. 6, 2016. https://www.youtube.com/watch?v=XovOYgeT2vk (Consulted June 20, 2021); "Interview at Cirm : David Ruelle," *Centre International de Rencontres Mathématiques Youtube Channel*, Jan. 25, 2016 https://www.youtube.com/watch?v=A4AgdKMyzbo (Consulted June 20, 2021).

[2] D. Ruelle, "A mathematical reformulation of Derrida's REM and GREM," *Commun. Math. Phys.* **108**, 225-239 (1987). https://doi.org/10.1007/BF01210613

work on the Sherrington-Kirkpatrick model, and Parisi's ansatz which proposed a solution for this model[3]. I tried unsuccessfully to make mathematical sense of the relevant physical literature.

Finally, I came to Derrida's papers[4], which I understood but appeared to me to contain a mathematical contradiction! I was brought to considering a Poisson distribution with density $\phi(\xi) = x\xi^{-x-1}$ for $\xi > 0$ with a constant $x \in (0,1)$. This is a simplified version of Poisson distributions considered by Derrida. The Poisson distribution with density ϕ corresponds to a decreasing sequence (ξn) of points on $\xi > 0$. One can see that in the average the sum $\sum \xi_n$ is $+\infty$ because $\int \xi\phi(\xi) \, d\xi = \infty$. Therefore it would seem that the sequence (ξn) cannot be normalized, killing the arguments of Derrida. A moment's reflection indicates however that the sequence (ξ_n) has almost certainly a maximum element and has thus a finite sum (because the integral $\int \xi\varphi(\xi) \, d\xi$ converges near 0). What happens is that $\sum \xi_n$ is almost surely finite, but has an infinite average. Once this is understood, the contents of [my paper] is just simple calculations.

PC: We heard from Michael Aizenman[5] that you gave a series of lectures at Rutgers about the REM/GREM/SK models in the mid-'80s. These lectures seem to have been very influential for US-based mathematical physicists. Can you give us some more context and details about these presentations?

DR: I don't remember well these lectures. They presumably reflected my current preoccupation with the Sherrington-Kirkpatrick model. Cf. the paper in Commun. Math. Phys. with Aizenman and Lebowitz[6], and the paper on REM and GREM [in Ref. 2].

PC: What led to the genesis of your work with Michael Aizenman and Joel Lebowitz on the high-temperature behavior of the SK model?

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[3] See, *e.g.*, G. Parisi, "Toward a mean field theory for spin glasses," *Phys. Lett. A* **73**, 203-205 (1979). https://doi.org/10.1016/0375-9601(79)90708-4; "Infinite number of order parameters for spin-glasses," *Phys. Rev. Lett.* **43**, 1754 (1979). https://doi.org/10.1103/PhysRevLett.43.1754; G. Parisi and G. Toulouse, "A Simple hypothesis for the spin glass phase of the infinite-ranged SK model," *J. Physique Lett.* **41**, 361-364 (1980). https://doi.org/10.1051/jphyslet:019800041015036100

[4] See, *e.g.*, B. Derrida, "Random-energy model: Limit of a family of disordered models," *Phys. Rev. Lett.* **45**,79- 82 (1980). https://doi.org/10.1103/PhysRevLett.45.79; B. Derrida and E. Gardner, "Solution of the generalised random energy model," *J. Phys. C* **19**, 2253 (1986). https://doi.org/10.1088/0022-3719/19/13/015

[5] P. Charbonneau, *History of RSB Interview: Michael Aizenman*, transcript of an oral history conducted 2021 by Patrick Charbonneau and Francesco Zamponi, History of RSB Project, CAPHÉS, École normale supérieure, Paris, 2021, ?? p. https://doi.org/XXXXXXXX

[6] M. Aizenman, J. L. Lebowitz and D. Ruelle, "Some rigorous results on the Sherrington-Kirkpatrick spin glass model," *Comm. Math. Phys.* **112**, 3–20 (1987). https://doi.org/10.1007/BF01217677

DR: The idea was that the complicated phase space structure portrayed by the Parisi Ansatz at low temperature was replaced by something simple at high temperature. Putting together what Aizenman, Lebowitz, and myself knew about correlations, we managed to control the structure of the high temperature phase of the Sherrington-Kirkpatrick model.

PC: Did you keep abreast of the later developments on the SK and related models? For instance, did you follow the relationship between the Parisi functional and the so-called Ruelle Probability Cascades?

DR: My contact with the later developments on the Parisi ansatz and probability cascades is essentially through the review by Erwin Bolthausen in the Séminaire Bourbaki, which describes the work of Parisi, Guerra, Talagrand, and Panchenko[7]. This review invites at a mathematical simplification of the arguments, but I have nothing to propose in this direction.

PC: In a 2016 interview[8], you hinted at your fascination with the conceptual foundation of the brain. Over the years, did you follow the development of neural networks, and their intimate relationship with spin glass models?

DR: I keep being fascinated by the brain, and have read some of the work by Lai-Sang Young[9] on the subject, but have not kept up with the work on neural networks.

PC: Did you teach about spin glasses and RSB elsewhere than at Rutgers? If yes, can you detail?

DR: I don't remember teaching about spin glasses except at Rutgers.

PC: Is there anything else you would like to share with us about this era that we may have missed?

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[7] Erwin Bolthausen, "Ultrametricity in Mean-Field Spin Glasses [after Dmitry Panchenko]," *Séminaire Bourmaki*, **66**(1082) (2014). https://smf.emath.fr/publications/expose-bourbaki-1082-sur-lultrametricite-dans-les-verres-de-spin; "Bourbaki - 29/03/14 - 2/4 - Erwin BOLTHAUSEN," *Institut Henri Poincaré Youtube Channel*, Mar. 31, 2014. https://www.youtube.com/watch?v=2s3y03q7mYo (Consulted June 20, 2021)
[8] See Ref. 1.
[9] Lai-Sang Young: https://en.wikipedia.org/wiki/Lai-Sang_Young

DR: I have collaborated with Enzo Marinari, Giorgio Parisi, and Paul Windey[10] on a relation between $1/f$ noise and random walks in random environments (not very convincing papers, no close relation with spin glasses[11]). A directly related paper with Henri Epstein[12]. This is an interdisciplinary paper which may contain a useful idea.

PC: Do you still have notes, papers, or correspondence from this time? If yes, do you intend to deposit them in an academic archive?

DR: I have correspondence from this period. The idea to deposit it in an academic archive sounds interesting, can you give me a reference?

PC: As a member of the Académie des Sciences, for instance, I'm pretty sure their archival service would welcome your material. I hope you get in touch with them. Thanks for your time and your enlightening answers.

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[10] Paul Windey (PhD, 1978) was then a postdoc at the Service de Physique Théorique. See: Paul Windey, *Schèmes de confinement linéaire des quarks,* PhD in Physics, Université libre de Bruxelles (1978). http://hdl.handle.net/2013/ULB-DIPOT:oai:dipot.ulb.ac.be:2013/214188

[11] E. Marinari, G. Parisi, D. Ruelle, and P. Windey, "Random walk in a random environment and $1/f$ noise," *Phys. Rev. Lett.* **50**, 1223 (1983). https://doi.org/10.1103/PhysRevLett.50.1223; E. Marinari, G. Parisi, D. Ruelle, and P. Windey, "On the interpretation of $1/f$ noise," Comm. Math. Phys. **89**, 1-12 (1983). https://doi.org/10.1007/BF01219521.

[12] H. Epstein and D. Ruelle, "Test of a probabilistic model of evolutionary success," *Phys. Rep.* **184**, 289-292 (1989). https://doi.org/10.1016/0370-1573(89)90048-3

Henri Epstein was (and remains) a *visiteur de longue durée* at IHÉS. See, *e.g.*, Allyn Jackson, "IHÉS at Forty," *Notices of the American Mathematical Society* **46**(3), 329-337 (1999). https://www.ams.org/notices/199903/index.html (Consulted June 20, 2021)